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Impulse Action on D-particles in Robertson-Walker Space Times, Higher-Order Logarithmic Conformal Algebras and Cosmological Horizons

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Abstract

We demonstrate that an impulse action ('recoil') on a D-particle embedded in a (four-dimensional) cosmological Robertson-Walker (RW) spacetime is described, in a σ-model framework, by a suitably extended higher-order logarithmic world-sheet algebra of relevant deformations. We study in some detail the algebra of the appropriate two-point correlators, and give a careful discussion as to how one can approach the world-sheet renormalization group infrared fixed point, in the neighborhood of which the logarithmic algebra is valid. It is found that, if the initial RW spacetime does not have cosmological horizons, then there is no problem in approaching the fixed point. However, in the presence of horizons, there are world-sheet divergences which imply the need for Liouville dressing in order to approach the fixed point in the correct way. A detailed analysis on the subtle subtraction of these divergences in the latter case is given. In both cases, at the fixed point, the recoil-induced spacetime is nothing other than a coordinate transformation of the initial spacetime into the rest frame of the recoiling D-particle. However, in the horizon case, if one identifies the Liouville mode with the target time, which expresses physically the back reaction of the recoiling D-particle onto the spacetime structure, it is found that the induced spacetime distortion results in the removal of the initial cosmological horizon and the eventual stopping of the acceleration of the Universe. In this latter sense, our model may be thought of as a conformal field theory description of a (toy) Universe characterized by a sort of 'phase transition' at the moment of impulse, implying a time-varying speed of light.

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1 Introduction and Summary

Placing D-branes in curved space times is not understood well at present. The main problem originates from the lack of knowledge of the complete dynamics of such solitonic objects. One would hope that such a knowledge would allow a proper study of the back reaction of such objects onto the surrounding space time geometry (distortion), and eventually a consistent discussion of their dynamics in curved spacetimes. Some modest steps towards an incorporation of curved space time effects in D-brane dynamics have been taken in the recent literature from a number of authors [1]. These works are dealing directly with world volume effects of D-branes and in some cases string dualities are used in order to discuss the effects of space time curvature.

A different approach has been adopted in [2, 3, 4, 5], in which we have attempted to approach some aspects of the problem from a world sheet view point, which is probably suitable for a study of the effects of the (string) excitations of the heavy brane. We have concentrated mainly on heavy D-particles, embedded in a *flat* target background space time. We have discussed the instantaneous action (impulse) of a 'force' on a heavy D-particle. The impulse may be viewed either as a consequence of 'trapping' of a *macrosopic number* of closed string states on the defect, and their eventual splitting into pairs of open strings, or, in a different context, as the result of a more general phenomenon associated with the *sudden* appearance of such defects. Our world sheet approach is a valid approximation only if one looks at times *long after* the event. Such impulse approximations usually characterize classical phenomena. In our picture we view the whole process as a *semi-classical* phenomenon, due to the fact that the process involves open string *recoil* excitations of the heavy D-particle, which are *quantum* in nature. It is this point of view that we shall adopt in the present article.

Such an approach should be distinguished from the problem of studying single-string scattering of a D-particle with closed string states in flat space times [6]. We have shown in [2, 3, 4, 5] that for a D-particle embedded in a d-dimensional *flat Minkowski* space time such an impulse action is described by a world-sheet σ-model deformed by appropriate 'recoil' operators, which obey a logarithmic conformal algebra [7]. The appearance of such algebras, which lie on the border line between conformal field theories and general renormalizable field theories in the two-dimensional world sheet, but can still be classified by conformal data, is associated with the fact that an impulse action (recoil) describes a *change* of the string/D-particle background, and as such it cannot be described by conformal symmetry all along. The *transition* between the two asymptotic states of the system before and (long) after the event is precisely described by deforming the associated σ-model by operators which *spoil* the conformal symmetry.

Indeed, the recoil operators are *relevant* from a world-sheet renormalization-group view point [3], and thus the induced string theory becomes non-critical, in need of Liouville dressing [8] in order to restore the conformal symmetry. The dressing results in the appearance of target-space metric distortion [5], which - under the identification of the Liouville mode with the time [9] - is interpreted as a backreaction of the recoiling D-particle defect onto the surrounding (initially flat) space time. Under such an impulse/recoil, there is in general an induced vacuum energy, which can even become time dependent [10]. Such time dependent vacuum energies in Cosmology have recently attracted a lot of attention as a challenge for string theory [11], given that in certain cases the corresponding Universes are characterized by cosmological horizons, and hence a field-theoretic S matrix cannot be defined for asymptotic states. From the point of view of Liouville string such a situation is expected [12], due to the fact that in Liouville strings, with the time identified with the Liouville mode [9], a scattering matrix cannot be defined.

In this work we shall attempt to extend the flat space time results of [2, 3, 5] to the physically relevant case of a Robertson-Walker (RW) cosmological background space time. Although, our results do not depend on the target space dimension, however, for definiteness we shall concentrate on the case of a D-particle embedded in a four-dimensional RW spacetime. It must be stressed that we shall not attempt here to present a complete discussion of the associated space time curvature effects, which - as mentioned earlier - is a very difficult task, still unresolved. Nevertheless, by concentrating on times much larger than the moment of impulse on the D-particle defect, one may ignore such effects to a satisfactory approximation. As we shall see, our analysis produces results which look reasonable and are of sufficient interest to initiate further research.

The vertex operators which describe the impulse in curved RW backgrounds obey a suitably extended (higher-order) logarithmic algebra. The algebra is valid at, and in the neighborhood of, a non-trivial infrared fixed point of the world-sheet Renormalization Group. For a RW spacetime of scale factor of the form t^p, where t is the target time, and $p > 1$ in the horizon case, the algebra is actually a set of logarithmic algebras up to order $[2p]$, which are classified by the appropriate higher-order Jordan blocks [7].

As in the flat case, which is obtained as a special limit of this more general case, the recoil deformations are relevant operators from a world-sheet Renormalization-Group viewpoint. One distinguishes two cases. In the first, the initial RW spacetime does not possess cosmological horizons. In this case it is shown that the limit to the conformal world-sheet non-trivial (infrared) fixed point can be taken smoothly without problems. On the other hand, in the case where the initial spacetime has cosmological horizons, such a limit is plagued by world-sheet divergences. These should be carefully subtracted in order to allow for a smooth approach to the fixed point. A detailed discussion of how this can be done is presented. In general, the divergences spoil the conformal invariance of the σ-model, thus implying the need for Liouville dressing [8] in order to properly restore the conformal symmetry.

Moreover, a careful discussion of the matching between the results of the Liouville dressing and those implied by the logarithmic algebra is given, which supports the possibility of identifying the world-sheet zero mode of the Liouville field (viewed as a local renormalization-group scale on the world sheet) with the target time. One distinguishes various cases which depend on whether the underlying theory lives in its critical dimension, and thus the only source of not criticality is the impulse action, or not. Such an identification induces target-space metric deformations, which are responsible for the *removal* of the cosmological horizon of the initial spacetime background, and the stopping of the acceleration of the Universe. Essentially the situation implies an effective time-dependent light velocity after the moment of impulse, which is responsible for the removal of the cosmological horizon. From this point of view our work may thus seem to provide a conformal-field-theory framework for a proper treatment of such time-varying speed of light scenaria [16] in the context of non-critical string theory [12].

2 Recoiling D-particles in Robertson-Walker Backgrounds

2.1 Geodesic Paths and Recoil

Let us consider a D-particle, located (for convenience) at the origin of the spatial coordinates of a four-dimensional space time, which at a time t_0 experiences an impulse. In a σ-model framework, the trajectory of the D-particle $y^i(t)$, i a spatial index, is described by inserting the following vertex operator

$$V = \int_{\partial\Sigma} G_{ij} y^j(t) \partial_n X^i \qquad (1)$$

where G_{ij} denotes the spatial components of the metric, $\partial\Sigma$ denotes the world-sheet boundary, ∂_n is a normal world-sheet derivative, X^i are σ-model fields obeying Dirichlet boundary conditions on the world sheet, and t is a σ-model field obeying Neumann boundary conditions on the world sheet, whose zero mode is the target time.

This is the basic vertex deformation which we assume to describe the motion of a D-particle in a curved geometry to leading order at least, where spacetime back reaction and curvature effects are assumed weak. Such vertex deformations may be viewed as a generalization of the flat-target-space case [13].

Perhaps a formally more desirable approach towards the construction of the complete vertex operator would be to start from a T-dual (Neumann) picture, where the deformation (1) should correspond to a proper Wilson loop operator of an appropriate gauge vector field. Such loop operators are by construction independent of the background geometry. One can then pass onto the Dirichlet picture by a T-duality transformation viewed as a canonical transformation from a σ-model viewpoint [14]. In principle, such a procedure would yield a complete form of the vertex operator in the Dirichlet picture, describing the path of a D-particle in a curved geometry. Unfortunately, such a procedure is not free from ambiguities at a quantum level [14], which are still unresolved for general curved backgrounds. Therefore, for our purposes here, we shall consider the problem of writing a complete form for the operator (1) in a RW spacetime background in the Dirichlet picture as an open issue. Nevertheless, for RW backgrounds at large times, ignoring

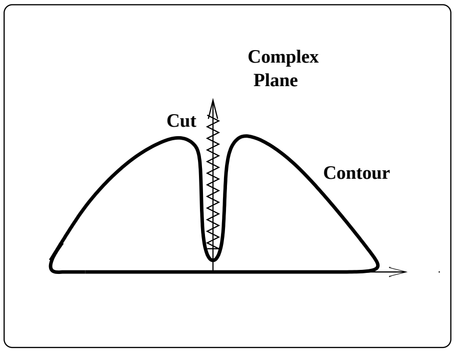

Figure 1: *Contour of integration in the complex plane to define the recoil operators $\mathcal{D}^{(q)}$, by proper treatment of the associated cuts.*

curvature effects proves to be a satisfactory approximation, and in such a case one may consider the vertex operator (1) as a sufficient description for the physical vertex operator of a D-particle. As we shall show below, the results of such analyses appear reasonable and interesting enough to encourage further studies along this direction.

For times long after the event, the trajectory $y^i(t)$ will be that of free motion in the curved space time under consideration. In the flat space time case, this trajectory was a straight line [13, 3, 4], and in the more general case here it will be simply the associated *geodesic*. Let us now determine its form, which will be essential in what follows.

The space time assumes the form:

$$ds^2 = -dt^2 + a(t)^2 (dX^i)^2 \tag{2}$$

where $a(t)$ is the RW scale factor. We shall work with expanding RW space times with scale factors

$$a(t) = a_0 t^p, \qquad p \in R^+ \tag{3}$$

The geodesic equations in this case read:

$$\begin{aligned} \ddot{t} + p t^{2p-1} (\dot{y}^i)^2 &= 0 \\ \ddot{y} + 2\frac{p}{t}(\dot{y}^i)\dot{t} &= 0 \end{aligned} \tag{4}$$

where the dot denotes differentiation with respect to the proper time τ of the D-particle.

With initial conditions $y^i(t_0) = 0$, and $dy^i/dt(t_0) \equiv v^i$, one easily finds that, for long times $t \gg t_0$ after the event, the solution acquires the form:

$$y^i(t) = \frac{v^i}{1-2p}\left(t^{1-2p} t_0^{2p} - t_0\right) + \mathcal{O}(t^{1-4p}), \qquad t \gg t_0 \tag{5}$$

To leading order in t, therefore, the appropriate vertex operator (1), describing the recoil of the D-particle, is:

$$V = \int_{\partial\Sigma} a_0^2 \frac{v^i}{1-2p}\Theta(t-t_0)\left(t t_0^{2p} - t_0 t^{2p}\right)\partial_n X^i \tag{6}$$

where $\Theta(t-t_0)$ is the Heaviside step function, expressing an instantaneous action (*impulse*) on the D-particle at $t = t_0$ [3, 5]. As we shall see later on, such deformed σ-models may be viewed as providing rather generic mathematical prototypes for models involving phase transitions at early stages of the Universe, leading effectively to time-varying speed of light. In the context of the present work, therefore, we shall be rather vague as far as the precise physical significance of the operator (6) is concerned, and merely exploit the consequences of such deformations for the expansion of the RW spacetime after time t_0, from both a mathematical and physical viewpoint.

In [3], we have studied the case $p = 0$, $a_0 = 1$, where the operators assumed the form $t\Theta_\epsilon(t)$ to leading order in t, where $\Theta_\epsilon(t)$ is the regulated form of the step function, given by [3]:

$$\Theta_\epsilon = -i\int_{-\infty}^{+\infty}\frac{d\omega}{2\pi}\frac{1}{\omega - i\epsilon}e^{i\omega\, t}, \qquad \epsilon \to 0^+ \tag{7}$$

As discussed in that reference, this operator forms a logarithmic pair [7] with $\epsilon\Theta_\epsilon(t)$, expressing physically fluctuations in the initial position of the D-particle.

In the current case, one may expand the integrand of (6) in a Taylor series in powers of $(t-t_0)$, which implies the presence of a series of operators, of the form $(t-t_0)^q\Theta_\epsilon(t-t_0)$, where q takes on the values $2p, 2p-1, \dots$, i.e. it is not an integer in general. In a direct generalization of the Fourier integral representation (7), we write in this case:

$$\mathcal{D}^{(q)} \equiv v_i(t-t_0)^q\Theta_\epsilon(t-t_0)\partial_n X^i = v_i\; N_q\int_{-\infty}^{+\infty}d\omega\frac{1}{(\omega - i\epsilon)^{q+1}}\;e^{i\omega(t-t_0)}\partial_n X^i\;,$$
$$N_q \equiv \frac{i^q}{\Gamma(-q)(1-e^{-i2\pi q})} = \frac{(-i)^{q+1}\Gamma(q+1)}{2\pi}\;, \tag{8}$$

where we have incorporated the velocity coupling v_i in the definition of the σ-model deformation, and we have defined the integral along the contour of figure 1, having chosen the cut to be from $+i\epsilon$ to $+i\infty$.

2.2 Extended Logarithmic world-sheet Algebra of recoil in RW backgrounds

Following the flat space time analysis of [3], we now proceed to discuss the conformal structure of the recoil operators in RW backgrounds. We shall do so by acting on the operator $D^{(q)}$ (8) with the world-sheet energy momentum tensor operator $T_{zz} \equiv T$ (in a standard notation). Due to the form of the background space time (2), the stress tensor T assumes the form

$$2T = -(\partial t)^2 + a^2(t)(\partial X^i)^2 \tag{9}$$

where, from now on, $\partial \equiv \partial_z$, unless otherwise stated. One can then obtain the relevant operator-product expansions (OPE) of T with the operators $\mathcal{D}^{(q)}$. For convenience in what follows we shall consider the action of each of the two terms in (9) on the operators $\mathcal{D}^{(q)}$ separately. For the first (time t-dependent part), one has, as $z \to w$:

$$-\frac{1}{2}(\partial t(z))^2\cdot\mathcal{D}^{(q)}(w) = \frac{v_i}{(z-w)^2}\left[N_q\int_{-\infty}^{+\infty}d\omega\frac{\omega^2/2}{(\omega-i\epsilon)^{q+1}}\;e^{i\omega\;t(w)}\right]\partial_n X^i =$$
$$\frac{1}{(z-w)^2}\left[-\frac{\epsilon^2}{2}\mathcal{D}^{(q)} + q\epsilon\mathcal{D}^{(q-1)} + \frac{q(q-1)}{2}\mathcal{D}^{(q-2)}\right] \tag{10}$$

The above formulæ were derived for asymptotically large time t, assuming the two-point correlators

$$\langle X^\mu(z)X^\nu(w)\rangle = 2G^{\mu\nu}\ln|z-w|^2 + \dots\;, \tag{11}$$

where the $\dots$ denote terms with negative powers of t, related to space-time curvature, which are subleading in the limit $t \to \infty$.

For the spatial part of (9) we consider the OPE $a(t(z))^2(\partial X^i(z))^2\mathcal{D}^{(q)}(w)$ as $z \to w$. Again, for convenience we shall do the time and space contractions separately:

$$t^{2p}(z)\cdot\mathcal{D}^{(q)}(w) = \int d\omega\tilde{\mathcal{D}}^{(q)}(\omega)t^{2p}(z)\cdot e^{i\omega(t(w)-t_0)} =$$
$$\int_0^\infty\frac{d\nu}{\Gamma(-2p)}\nu^{-1-2p}\int d\omega\tilde{\mathcal{D}}^{(q)}(\omega)e^{-\nu t(z)}\cdot e^{i\omega(t(w)-t_0)} \tag{12}$$

Using the OPE $e^{-\nu t(z)} \cdot e^{-i\omega(t(w)-t_0)} \sim |z-w|^{i\nu\omega} e^{-\nu t(z) - i\omega(t(z)-t_0) + \mathcal{O}(z-w)}$ one obtains (as $z \sim w$):

$$
\begin{aligned}
& t(z)^{2p} \cdot \mathcal{D}^{(q)}(w) = \int_0^\infty \frac{d\nu}{\Gamma(-2p)} \nu^{-1-2p} e^{-\nu t(z)} \mathcal{D}^{(q)}(t - t_0 - \nu \ln|z-w|) = \\
& t^{2p} \int_0^\infty \frac{d\nu}{\Gamma(-2p)} \nu^{-1-2p} e^{-\nu} \mathcal{D}^{(q)}(t - t_0 - \frac{\nu}{t} \ln|z-w|) = \\
& t^{2p} \left[\mathcal{D}^{(q)}(t-t_0) - \frac{1}{t} \ln|z-w| \frac{\Gamma(1-2p)}{\Gamma(-2p)} \frac{d}{dt} \mathcal{D}^{(q)}(t-t_0) + \mathcal{O}(t-t_0)^{q-2} \right]
\end{aligned} \tag{13}
$$

We now observe that $\frac{d\mathcal{D}^{(q)}}{dt} = q\mathcal{D}^{(q-1)} - \epsilon\, \mathcal{D}^{(q)}$, where both terms have vacuum expectation values of the same order in ϵ, as we shall see below, and hence both should be kept in our perturbative expansion.

Expanding the various terms around t_0, $t^s = (t-t_0)^s + s\, t_0 (t-t_0)^{s-1} + \frac{t_0^2}{2}(s)(s-1)(t-t_0)^{s-2} + \mathcal{O}([t-t_0]^{s-3})$, one has:

$$
\begin{aligned}
& t^{2p}(z) \cdot \mathcal{D}^{(q)}(w) = \mathcal{D}^{(2p+q)}(t-t_0) + (2p\, t_0 - 2p\, \epsilon \ln|z-w|)\ \mathcal{D}^{(2p+q-1)} + \\
& + \left(\frac{t_0^2}{2} 2p(2p-1) + [2pq + (2p - 4p^2)\epsilon\, t_0] \ln|z-w| \right) \mathcal{D}^{(2p+q-2)}(t-t_0) + \\
& + \mathcal{O}([t-t_0]^{2p+q-3})
\end{aligned} \tag{14}
$$

where it is worthy of mentioning that inside the subleading terms there are higher logarithms of the form $\ln^n|z-w|$, where $n = 2, 3, 4, \ldots$.

We now come to the OPE between the spatial parts. In view of (11), upon expressing ∂_z in normal ∂_n and tangential parts, and imposing Dirichlet boundary conditions on the world-sheet boundary where the operators live on, we observe that such operator products take the form:

$$
(\partial X^j(z))^2 \cdot \partial_n X^i(w) \sim G^{ii} \frac{1}{(z-w)^2} \partial_n X^i \sim \frac{t^{-2p}}{(z-w)^2} \partial_n X^i, \qquad (\text{no sum over } i) \tag{15}
$$

Performing the last contraction with the t^{-2p}, following the previous general formulæ and collecting appropriate terms, one obtains:

$$
\begin{aligned}
& T(z) \cdot \mathcal{D}^{(q)}[(t-t_0)(w)] = \frac{1 - \frac{\epsilon^2}{2}}{(z-w)^2} \mathcal{D}^{(q)}[(t-t_0)(w)] + \frac{q\epsilon}{(z-w)^2} \mathcal{D}^{q-1}[(t-t_0)(w)] + \\
& + \frac{\frac{q(q-1)}{2} - 2p^2 \ln|z-w| - 2p^2\, \epsilon^2 \ln^2|z-w|}{(z-w)^2} \mathcal{D}^{(q-2)}[(t-t_0)(w)] + \mathcal{O}([t-t_0]^{q-3})
\end{aligned} \tag{16}
$$

where again inside the subleading terms there are higher logarithms.

We next notice that, as a consistency check of the formalism, one can calculate the OPE (16) in case one considers matrix elements between *on-shell* physical states. In the context of σ-models, we are working with, the physical state condition implies the constraint of the vanishing of the world-sheet stress-energy tensor $2T = -(\partial t)^2 + a(t)^2 (\partial X^i)^2 = 0$. This condition allows $(\partial X^i)^2$ to be expressed in terms of $(\partial t)^2$, which is consistent even at a correlation function level in the case of very target times $t \gg t_0$, since in that case, the correlator $\langle X^i t \rangle$ is subleading, as mentioned previously. Implementing this, it can be then seen that the OPE between the spatial parts of T and $\mathcal{D}^{(q)}$ is:

$$
\begin{aligned}
& a^2(t)(\partial X^i)^2 \cdot \mathcal{D}^{(q)} = t^{-2p} (\partial t)^2 \cdot \{ \mathcal{D}^{(2p+q)}(t-t_0) + (2p\, t_0 - 2p\, \epsilon \ln|z-w|)\ \mathcal{D}^{(2p+q-1)} + \\
& + \left(\frac{t_0^2}{2} 2p(2p-1) + [2pq + (2p-4p^2)\epsilon\, t_0] \ln|z-w| \right) \mathcal{D}^{(2p+q-2)}(t-t_0) + \\
& + \mathcal{O}([t-t_0]^{2p+q-3}) \}.
\end{aligned} \tag{17}
$$

Performing the appropriate contractions, and adding to this result the OPE of the temporal part of T with $\mathcal{D}^{(q)}$, i.e. the quantity $-\frac{\epsilon^2}{2} \mathcal{D}^{(q)} + q\epsilon \mathcal{D}^{(q-1)} + \frac{1}{2} q(q-1) \mathcal{D}^{(q-2)}$, we obtain:

$$
\begin{aligned}
& T \cdot \mathcal{D}^{(q)}|_{\text{on-shell}} = \left(-2p\epsilon - pt_0\, \epsilon^2 + p\epsilon^2 \ln(a/L) \right)\ \mathcal{D}^{(q-1)} + \\
& + \{ t_0^2 \epsilon^2\ 2p(2p+1) - 3\epsilon^2 p(2p+q) \ln(a/L) - 2\epsilon^3 (p+p^2) t_0 \ln(a/L) - 2p^2 \epsilon^4 \ln^2(a/L) + \\
& + \epsilon(2p+q) 2p\, t_0 - (4p^2 + 4pq - 2p) \} \mathcal{D}^{(q-2)} + \mathcal{O}([t-t_0]^{q-3}))
\end{aligned} \tag{18}
$$

From the above we observe that the on-shell operators become marginal as they should, given that an on-shell theory ought to be conformal. Moreover, and more important, the world-sheet divergences *disappear* upon imposing the condition

$$\epsilon^2 \ln(L/a)^2 = \xi_0 = \text{constant independent of } \epsilon,\ a,\ L \tag{19}$$

where L (a) is the world-sheet (ultraviolet) infrared cut-off on the world sheet. As we shall discuss later on, this condition will be of importance for the closure of the logarithmic algebra, which characterizes the fixed point [3]. Hence, the conformal invariance is preserved by the on-shell states, any dependence from it being associated with *off-shell* states.

We next notice that, in the context of the RW metric (2), there are two cases of expanding universes, one corresponding to $0 < p \leq 1$, and the other to $p > 1$. Whenever $p \leq 1$ (which notably incorporates the cases of both radiation and matter dominated Universes) there is *no horizon*, given that the latter is given by:

$$\delta(t) = a(t) \int_{t_0}^{\infty} \frac{dt'}{a(t')} \tag{20}$$

In this case the relevant value for q is $q = 2p \leq 2$. On the other hand, for the case $p > 1$, i.e. $q > 2$ there is a non-trivial cosmological *horizon*, which as we shall see requires special treatment from a conformal symmetry viewpoint.

We commence with the no-horizon case, $1 < q \leq 2$. We first notice that the linear in t term in (6) leads to the conventional logarithmic algebra, discussed in [3], corresponding to a pair of impulse ('recoil') operators C, D. The main point of our discussion below is a study of the t^{2p} terms in (6), and their connection to other logarithmic algebras. Indeed, we observe that a logarithmic algebra [7, 2, 3] can be obtained for these terms of the operators, if we define $\mathcal{D} \equiv \mathcal{D}^{(q)}$ and $\mathcal{C} \equiv q\epsilon\mathcal{D}^{(q-1)}$. In this case we have the following OPE with T:

$$\begin{aligned}
(z-w)^2\ T \cdot \mathcal{D} &= (1 - \frac{\epsilon^2}{2})\mathcal{D} + \mathcal{C}, \\
(z-w)^2\ T \cdot \mathcal{C} &= (1 - \frac{\epsilon^2}{2})\mathcal{C} + \mathcal{O}([t-t_0]^{q-2}),
\end{aligned} \tag{21}$$

where throughout this work we ignore terms with negative powers in $t - t_0$ (e.g. of order $q - 2$ and higher), for large $t \gg t_0$. Notice that in the case $q < 1$ (i.e. $p < 1/2$) the $\mathcal{C}$ operator defined above is absent.

In the second case $p > 1$ one faces the problem of having cosmological horizons (cf. (20)), which recently has attracted considerable attention in view of the impossibility of defining a consistent scattering S-matrix for asymptotic states [11, 12]. In this case the operator $\mathcal{D}^{(q-2)}$ is *not subleading* and one has an *extended (higher-order) logarithmic algebra* defined by (16). It is interesting to remark that now the logarithmic world-sheet terms in the coefficient of the $\mathcal{D}^{(q-2)}$ operator imply that the limit $z \to w$ is plagued by ultraviolet world-sheet divergences, and hence the world-sheet conformal invariance is spoiled. This necessitates Liouville dressing, in order to restore the conformal symmetry [8]. As we shall show later, in such cases with horizons the recoil of the D-particle may induce a non-trivial backreaction on the spacetime geometry, which results in an effective spacetime in which the horizons *disappear*. This happens, as we shall discuss later, in the context of Liouville strings with the identification of the Liouville mode with time.

We now turn to a study of the correlators of the various $\mathcal{D}^{(q)}$ operators, which will complete the study of the associated logarithmic algebras, in analogy with the flat target-space case of [3]. From the algebra (16) we observe that we need to evaluate correlators between $\mathcal{D}^{(q)}, \mathcal{D}^{(q-n)},\ n = 0, 1, 2, \ldots$. We shall evaluate correlators $\langle \ldots \rangle$ with respect to the free world-sheet action, since we work to leading order in the (weak) coupling v_i. For convenience below we shall restrict ourselves only to the time-dependent part of the operators $\mathcal{D}$. The incorporation of the $\partial_n X^i$ is trivial, and will be implied in what follows. With these in mind one has:

$$\langle \mathcal{D}^{(q)}(z)\mathcal{D}^{(q-n)}(w)\rangle = N_q N_{q-n} \int \int_{-\infty}^{+\infty} \frac{d\omega d\omega'}{(\omega - i\epsilon)^{q+1}(\omega' - i\epsilon)^{q-n+1}} \langle e^{-i\omega t(z)}\ e^{-i\omega' t(w)}\rangle \tag{22}$$

where $\epsilon \to 0^+$. As already mentioned, we work to leading order in time $t \gg \infty$, and hence we can we apply the formula (11) for two-point correlators of the X^μ fields to write [1]

$$
\begin{aligned}
&\langle e^{-i\omega t(z)}\ e^{-i\omega' t(w)}\rangle = e^{-\frac{\omega^2}{2}\langle t(z)t(z)\rangle - \frac{\omega'^2}{2}\langle t(w)t(w)\rangle - \omega\omega'\langle t(z)t(w)\rangle} = \\
&= e^{-(\omega+\omega')^2 \ln(L/a)^2 + 2\omega\omega' \ln(|z-w|/a)^2}\ , \qquad (23)
\end{aligned}
$$

where we took into account that $\mathrm{Lim}_{z\to w}\ \langle t(z)t(w)\rangle = -2\ln(a/L)^2$. Given that $\ln(L/a)$ is very large, one can approximate $e^{-(\omega+\omega')^2\ln(L/a)^2} \simeq \frac{\sqrt{\pi}}{\sqrt{\ln(L/a)^2}}\delta(\omega+\omega')$. Thus we obtain:

$$
\langle \mathcal{D}^{(q)}(z)\mathcal{D}^{(q-n)}(w)\rangle = (-1)^{-q+n-1}\ N_q\ N_{q-n}\mathcal{J}^{(q)}_n\ , \qquad \mathcal{J}^{(q)}_n \equiv \sqrt{\frac{\pi}{\alpha}}\int_{-\infty}^{+\infty} \frac{d\omega\ e^{-\omega^2\lambda}\ (\omega+i\epsilon)^n}{(\omega^2+\epsilon^2)^{q+1}} \qquad (24)
$$

where $\lambda \equiv 2\ln(|z-w|/a)^2$, and $\alpha \equiv \ln(L/a)^2$.

Below, for definiteness, we shall be interested in the case $2 < q < 3$, in which the relevant correlators are given by $n = 0, 1, 2$. One has:

$$
\begin{aligned}
\mathcal{J}^{(q)}_0\ &= \sqrt{\frac{\pi}{\alpha}}\ \epsilon^{-2q-1}\ f_q(\epsilon^2\lambda)\ ; \\
f_q(\xi) &= \sqrt{\pi}\frac{\Gamma(\frac{1}{2}+q)}{\Gamma(1+q)}\ F(\frac{1}{2},\ \frac{1}{2}-q\ ;\xi) +\ \xi^{\frac{1}{2}+q}\ \Gamma(-\frac{1}{2}-q)\ F(1+q,\ \frac{3}{2}+q\ ;\xi) \\
\mathcal{J}^{(q)}_1\ &=\ i\epsilon\mathcal{J}^{(q)}_0\ , \\
\mathcal{J}^{(q)}_2 &= -2\epsilon^2\ \mathcal{J}^{(q)}_0 + \mathcal{J}^{(q-1)}_0 = -\frac{\partial}{\partial\lambda}\mathcal{J}^{(q)}_0 - \epsilon^2\mathcal{J}^{(q)}_0 \qquad (25)
\end{aligned}
$$

where $F(a,b;z) = 1 + \frac{a}{b}\frac{z}{1!} + \frac{a(a+1)}{b(b+1)}\frac{z^2}{2!} + \ldots$ is the degenerate (confluent) hypergeometric function. Thus, the form of the algebra away from the fixed point ('*off-shell form*'), i.e. for $\epsilon^2 \neq 0$, is:

$$
\begin{aligned}
&\langle \mathcal{D}^{(q)}(z)\mathcal{D}^{(q)}(0)\rangle = \tilde{N}_q^2\ \sqrt{\frac{\pi}{\xi_0}}\ \Big(f_q(2\xi_0)(\frac{\alpha}{\xi_0})^q + 2\ f'_q(2\xi_0)(\frac{\alpha}{\xi_0})^{q-1}\ \ln(|z/L|^2) + \\
&+\frac{1}{2}\ f''_q(2\xi_0)(\frac{\alpha}{\xi_0})^{q-2}\ 4\ln^2(|z/L|^2) + \mathcal{O}(\alpha^{q-3})\Big)\ , \\
&\epsilon\ q\ \langle \mathcal{D}^{(q)}(z)\mathcal{D}^{(q-1)}(0)\rangle = \tilde{N}_q^2\sqrt{\frac{\pi}{\xi_0}}\ \Big(f_q(2\xi_0)(\frac{\alpha}{\xi_0})^{q-1} + \\
&+2\ f'_q(2\xi_0)(\frac{\alpha}{\xi_0})^{q-2}\ \ln(|z/L|^2) + \mathcal{O}(\alpha^{q-3})\Big)\ , \\
&\epsilon^2\ q^2\ \langle \mathcal{D}^{(q-1)}(z)\mathcal{D}^{(q-1)}(0)\rangle = \tilde{N}_q^2\ \sqrt{\frac{\pi}{\xi_0}}\ f_{q-1}(2\xi_0)(\frac{\alpha}{\xi_0})^{q-2} + \mathcal{O}(\alpha^{q-3}), \\
&\epsilon^2\ q\ (q-1)\ \langle \mathcal{D}^{(q)}(z)\mathcal{D}^{(q-2)}(0)\rangle = -\tilde{N}_q^2\ \sqrt{\frac{\pi}{\xi_0}}\left(f_q(2\xi_0) + f'_q(2\xi_0)\right)\ (\frac{\alpha}{\xi_0})^{q-2} + \mathcal{O}(\alpha^{q-3})\ , \\
&\epsilon^3\ q^2(q-1)\ \langle \mathcal{D}^{(q-1)}(z)\mathcal{D}^{(q-2)}(0)\rangle = \mathcal{O}(\alpha^{q-3})\ , \\
&\epsilon^4\ q^2(q-1)^2\ \langle \mathcal{D}^{(q-2)}(z)\mathcal{D}^{(q-2)}(0)\rangle = \mathcal{O}(\alpha^{q-4}) \qquad (26)
\end{aligned}
$$

where $\tilde{N}_q = \frac{\Gamma(1+q)}{2\pi}$, and ξ_0 has been defined in (19).

Notice that the above algebra is plagued by world-sheet ultraviolet divergences as $\epsilon^2 \to 0^+$, thereby making the approach to the fixed (conformal) point subtle. As becomes obvious from (19), the non-trivial fixed point $\epsilon \to 0^+$ corresponds to $L/a \to +\infty$, i.e. it is an infrared world-sheet fixed point. In order to understand the approach to the infrared fixed point, it is important to make a few remarks first, motivated by physical considerations.

From the integral expression of the regularized Heaviside function [3] (7) it becomes obvious that a scale $1/\epsilon$ for the target time is introduced. This, together with the fact that the scale ϵ is

[1] Here we use simplified propagators on the boundary, with the latter represented by a straight line; this means that the arguments of the logarithms are real [3]. To be precise, one should use the full expression for the propagator on the disc, along the lines of [4]. As shown there, and can be checked here as well, the results are unaffected.

connected (19) to the world renormalization-group scales L/a, implies naturally the introduction of a 'renormalized' σ-model coupling/velocity $v_{R,i}(\frac{1}{\epsilon})$ at the scale $\frac{1}{\epsilon}$:

$$v_{R,i}(\frac{1}{\epsilon}) \sim \left(\frac{1}{\epsilon}\right)^{q-1} \tag{27}$$

for a trajectory $y_i(t) \sim t^q$. This normalization would imply the following rescaling of the operators

$$\mathcal{D}^{(q-n)} \to \epsilon^{q-1}\mathcal{D}^{(q-n)} \tag{28}$$

As a consequence, the factors $\epsilon^{2(1-q)}$ in (25), (26) are removed. In the context of the world-sheet field theory this renormalization can be interpreted as a subtraction of the ultraviolet divergences by the addition of appropriate counterterms in the σ model.

The approach to the infrared fixed point $\epsilon \to 0^+$ can now be made by looking at the *connected* two point correlators between the operators $\mathcal{D}^{(q)}$ defined by

$$\langle \mathcal{A}\mathcal{B}\rangle_c = \langle \mathcal{A}\mathcal{B}\rangle - \langle \mathcal{A}\rangle\ \langle \mathcal{B}\rangle\ , \tag{29}$$

where the one-point functions are given by:

$$\begin{aligned} \langle \mathcal{D}^{(s)}\rangle &= N_s \int \frac{d\omega}{(\omega - i\epsilon)^{s+1}}\langle e^{i\omega t}\rangle = N_s \int \frac{d\omega}{(\omega - i\epsilon)^{s+1}} e^{-\omega^2\alpha} = \tilde{N}_s\ \epsilon^{-s}\ h_s(\epsilon^2\alpha)\ , \\ h_s(x) &= -\frac{x^{s/2}}{2}\left(\frac{4\pi}{\Gamma(\frac{1+s}{2})}\ \sqrt{\pi}\ F(1+\frac{s}{2}\ ,\frac{3}{2}\ ,x) - \frac{2\pi}{\Gamma(1+\frac{s}{2})}\ F(\frac{1+s}{2}\ ,\frac{1}{2}\ ,x)\right). \end{aligned} \tag{30}$$

For the two-point function of the $\mathcal{D}^{(q)}$ operator the result is:

$$\langle \mathcal{D}^{(q)}(z)\mathcal{D}^{(q)}(0)\rangle_c = \tilde{N}_q\epsilon^{-2}\left(\frac{\sqrt{\pi}}{\xi_0} f_q(2\xi_0 + 2\epsilon^2 \ln|z/L|^2) - h_q^2(\xi_0)\right)\ . \tag{31}$$

Expanding in powers of ϵ, we obtain

$$\begin{aligned} \langle \mathcal{D}^{(q)}(z)\mathcal{D}^{(q)}(0)\rangle_c &= \tilde{N}_q\epsilon^{-2}\left(\frac{\sqrt{\pi}}{\sqrt{\xi_0}} f_q(2\xi_0) - h_q^2(\xi_0)\right) + \tilde{N}_q^2\ \frac{\sqrt{\pi}}{\sqrt{\xi_0}}\ f_q'(2\xi_0)2\ln|z/L|^2 + \\ &+ \epsilon^2\tilde{N}_q^2\sqrt{\frac{\pi}{\xi_0}}\frac{1}{2} f_q''(2\xi_0)4\ln^2|z/L|^2 + \ldots \end{aligned} \tag{32}$$

where $\ldots$ denote terms that vanish as $\epsilon \to 0^+$.

To avoid the divergences coming from the ϵ^{-2} factors, the following condition must be satisfied: there must be a solution $\xi_0 = \xi_0(q)$ of the equation: $\mathcal{H}(\xi_0) \equiv \frac{\sqrt{\pi}}{\sqrt{\xi_0}} f_q(2\xi_0) - h_q^2(\xi_0) = 0$. The existence of such a solution can be verified numerically (see figure 2). Analytically this can be confirmed by looking at the asymptotic behaviour of the function $\mathcal{H}(x)$ as $x \to \infty$, which yields a negative value: $\mathcal{H}(x \to \infty) \sim -\frac{\pi^3 x^{2q} e^{2x}}{\Gamma^2(\frac{1+q}{2})\Gamma^2(1+\frac{q}{2})} < 0$. This behaviour comes entirely from the term $h_q^2(x)$, given that $f_q(x \to \infty) \to 0^+$.

As we shall show below, for various values of q, near the fixed point $\epsilon \to 0^+$, one can construct higher order logarithmic algebras, whose highest power is determined by the dominant terms in the operator algebra of correlators (26), (21). To this end, we first remark that in the above analysis we have dealt with a small but otherwise arbitrary parameter ϵ, which allows us to keep as many powers as required by (26) in conjunction with the value of q. The value of ϵ determines the distance from the fixed point.

For $1 < q < 2$, there are only two dominant operators as the time $t \to \infty$, $\mathcal{D}$,$\mathcal{C}$. In this case one obtains a conventional logarithmic conformal algebra of two-point functions near the fixed point:

$$\begin{aligned} &\langle \mathcal{D}^{(q)}(z)\ \mathcal{D}(0)^{(q)}\rangle_c = \langle \mathcal{D}(z)\ \mathcal{D}(0)\rangle_c \sim \tilde{N}_q^2\ \frac{\sqrt{\pi}}{\sqrt{\xi_0}}\ f_q'(2\xi_0)2\ln|z/L|^2\ , \\ &\epsilon\ q\ \langle \mathcal{D}^{(q-1)}(z)\ D^{(q)}(0)\rangle_c = \langle \mathcal{C}(z)\ \mathcal{D}(0)\rangle_c \sim \tilde{N}_q^2\ \left(h_q^2(\xi_0) - h_{q-1}\ h_q(\xi_0)\right)\ , \end{aligned} \tag{33}$$

and all the other correlators are subleading as $t \to \infty$.

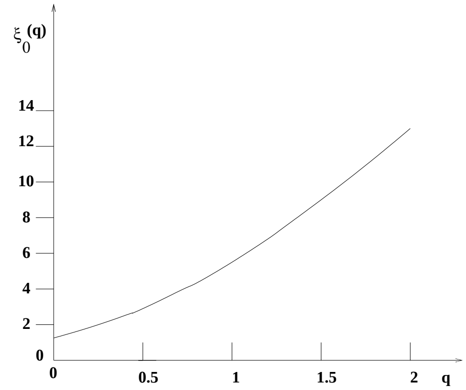

Figure 2: *Graphic solution of the equation* $\frac{\sqrt{\pi}}{\sqrt{\xi_0}} f_q(2\xi_0) - h_q^2(\xi_0) = 0.$

Therefore, the *on shell algebra* is of the conventional *logarithmic form* [7], between a pair of operators, and hence, $\mathcal{D}^{(q-2)}$ and subsequent operators, which owe their existence to the non-trivial RW metric, do not modify the two-point correlators of the standard logarithmic algebra of 'recoil' (impulse) [3] [2].

Next, we consider the case where $2 < q < 3$. In this case, from (26) we observe that there are now three operators which dominate in the limit $t \to \infty$, $\mathcal{D}$, $\mathcal{C}$ and $\mathcal{B} = \epsilon^2\ q\ (q-1)\mathcal{D}^{(q-2)}$, whose form is implied from (21), in analogy with $\mathcal{C}$. The corresponding algebra of correlators consists of parts forming a conventional logarithmic algebra, and parts forming a second-order logarithmic algebra, the latter being obtained from terms of order ϵ^2 in the appropriate two-point connected correlators (cf. (32) *etc.*), which are denoted by a superscript $\langle \dots \rangle_c^{(2)}$:

$$\begin{aligned}
\langle \mathcal{D}(z)\ \mathcal{D}(0)\rangle_c^{(2)} &= \tilde{N}_q^2 \sqrt{\frac{\pi}{\xi_0}} \frac{1}{2} f_q''(2\xi_0) 4\ln^2|z/L|^2\ , \\
\langle \mathcal{C}(z)\ \mathcal{D}(0)\rangle_c^{(2)} &= \tilde{N}_q^2 \sqrt{\frac{\pi}{\xi_0}} 2 f_q'(2\xi_0) \ln|z/L|^2\ , \\
\langle \mathcal{C}(z)\ \mathcal{C}(0)\rangle_c^{(2)} &= \tilde{N}_q^2 \sqrt{\frac{\pi}{\xi_0}} f_{q-1}(2\xi_0)\ , \\
\langle \mathcal{B}(z)\ \mathcal{D}(0)\rangle_c^{(2)} &= -\tilde{N}_q^2 \sqrt{\frac{\pi}{\xi_0}} \left(f_q(2\xi_0) + f_q'(2\xi_0)\right)\ , \\
\langle \mathcal{C}(z)\ \mathcal{B}(0)\rangle_c^{(2)} &= \langle \mathcal{B}(z)\ \mathcal{B}(0)\rangle_c^{(2)} = 0
\end{aligned} \tag{34}$$

where the last two correlators are of order ϵ^4 and ϵ^6 respectively, that is of higher order than the ϵ^2 terms, and hence they are viewed as zero to the order we are working here.

In general, if one considers $q > 3$ one arrives at higher order logarithmic algebras [7], with the highest power given by the integer value of q, $[q]$. This is an interesting feature of the recoil-induced motion of D-particles in RW backgrounds with scale factors $\sim t^p$, $p > 1$, corresponding to cosmological horizons and accelerating Universes. In such a case the order of the logarithmic algebra is given by $[2p]$. It is interesting to remark that radiation and matter (dust) dominated RW Universes would imply simple logarithmic algebras.

We now notice that, under a world-sheet finite-size scaling,

$$L \to L' = L\ e^{\mathcal{T}\,\mathcal{K}(q)}\ , \qquad \epsilon^{-2} \to (\epsilon')^{-2} = \epsilon^{-2} + \mathcal{T} \tag{35}$$

with $\mathcal{K}(q)$ a function of q determined by (33), the operators $\mathcal{C},\ \mathcal{D}, \dots$, and consequently the target-time t, transform in a non trivial way. In particular, for t one has:

$$\left(\frac{\epsilon'}{\epsilon}\right)^{q-1} \mathcal{Z}\ (\mathcal{T})^q t(\mathcal{T})^q = t^q + q\ \epsilon\ \mathcal{T}\ t^{q-1} + \mathcal{O}(\epsilon^2) \tag{36}$$

[2]We note at this stage that, in our case of non-trivial cosmological RW spacetimes, the pairs of operators $\mathcal{D},\mathcal{C}$ do not represent velocity and position as in the flat space time case of ref. [3], but rather velocity and acceleration. This implies that, under a finite-size scaling of the world sheet, the induced transformations of these operators do not form a representation of the Galilean transformations of the flat-space-time case.

where $\mathcal{Z}(\mathcal{T})$ is a wave function renormalization of the world-sheet field $t(z)$, which can be chosen in a natural way so that $\left(\frac{\epsilon'}{\epsilon}\right)^{q-1} \mathcal{Z}(\mathcal{T})^q = 1$. This implies

$$\begin{aligned} t(\mathcal{T})^q &= (t + \epsilon\mathcal{T})^q + \mathcal{O}(\epsilon^2) \ , \\ t(\mathcal{T}) &= t + \epsilon\mathcal{T} + \mathcal{O}(\epsilon^2) \ , \end{aligned} \tag{37}$$

i.e. that a shift in the target time is represented as $\epsilon\,\mathcal{T}$. Of course, at the fixed point, $\epsilon = 0$, the field $t(z)$ does not run, as expected. As we shall see in the next section, the shift (37) is consistent with the identification of the Liouville mode with the target time, in case one wishes to discuss certain aspects of slightly off-shell string physics.

3 Space Time Metrics

3.1 Vertex Operator for the Path and associated SpaceTime Geometry

In this section we shall discuss the implications of the world-sheet deformation (1) for the spacetime geometry. In particular, we shall show that its rôle is to preserve the Dirichlet boundary conditions on the X^i by changing coordinate system, which is encoded in an induced change in the space time geometry G_{ij}. The final coordinates, then, are coordinates in the rest frame of the recoiling particle, which naturally explains the preservation of the Dirichlet boundary condition.

To this end, we first rewrite the world-sheet boundary vertex operator (1) as a bulk operator:

$$\begin{aligned} V &= \int_{\partial\Sigma} G_{ij} y^j(t) \partial_n X^i = \int_{\Sigma} \partial_\alpha \left(y_i(t) \partial^\alpha X^i \right) = \\ &= \int_{\Sigma} \left(\dot{y}_i(t) \partial_\alpha t \partial^\alpha X^i + y_i \partial^2 X^i \right) \end{aligned} \tag{38}$$

where the dot denotes derivative with respect to the target time t, and α is a world-sheet index. Notice that it is the covariant vector y_i which appears in the formula, which incorporates the metric G_{ij}, $y_i = G_{ij} y^j$.

To determine the background geometry, which the string is moving in, it is sufficient to use the classical motion of the string, described by the world-sheet equations:

$$\partial^2 X^i + \Gamma^i{}_{\mu\nu} \partial_\alpha X^\mu \partial^\alpha X^\nu = 0, \tag{39}$$

where μ, ν are space time indices, $\alpha = 1, 2$ is a world-sheet index, ∂^2 is the Laplacian on the world sheet, and i is a target spatial index.

The relevant Christoffel symbol in our RW background case, is $\Gamma^i{}_{ti}$, and thus the operator (38) becomes:

$$\int_{\Sigma} \left(\dot{y}_i - 2 y_i(t) \Gamma^i{}_{ti} \right) \partial_\alpha t \partial^\alpha X^i \tag{40}$$

from which we read an induced non-diagonal component for the space time metric

$$2 G_{0i} = \dot{y}_i - 2 y_i(t) \Gamma^i{}_{ti} \tag{41}$$

In the RW background (2) the path $y_i(t)$ is described (5) by (notice again we work with covariant vector y_i):

$$y_i(t) = \frac{v_i a_0^2}{1 - 2p} \left(t t_0^{2p} - t_0 t^{2p} \right) \tag{42}$$

which gives $2G_{0i} = a^2(t_0) v_i$, yielding for the metric line element:

$$ds^2 = -dt^2 + v_i a^2(t_0) dt dX^i + a^2(t)(dX^i)^2, \qquad \text{for} \qquad t > t_0 \tag{43}$$

As expected, this spacetime has precisely the form corresponding to a Galilean-boosted frame (the D-particle's rest frame), with the boost occurring suddenly at time $t = t_0$.

This can be understood in a general fashion by first noting that (41) can be written in a general covariant form as:

$$2G_{0i} = \nabla_t y_i \quad (= \nabla_t y_i + \nabla_i t) \tag{44}$$

which is the general coordinate transformation associated with y_i from a passive (Lie derivative) point of view.

In general, given the boundary condition $\partial_n t = 0$, one can write the operator (1), in a covariant form by expressing it as a world-sheet bulk operator:

$$V = \int_{\partial\Sigma} y_\mu \partial_n X^\mu = \int_\Sigma \partial_\alpha \left(y_\mu \partial^\alpha X^\mu \right) = \int_\Sigma \nabla_\mu y_\nu \partial_\alpha X^\mu \partial^\alpha X^\nu \tag{45}$$

where in the last step, we have used again the string equations of motion (39). From this expression, one then derives the induced change in the metric

$$2\delta G_{\mu\nu} = \nabla_\mu y_\nu + \nabla_\nu y_\mu \tag{46}$$

which is the familiar expression of the Lie derivative under the coordinate transformation associated with y_μ.

In all the above expressions we have taken the limit $\epsilon \to 0$, which corresponds to considering the ratio of world-sheet cut-offs $a/L \to 0$, implying that one approaches the infrared fixed point in a Wilsonian sense. As noted previously, in the context of the logarithmic conformal analysis of the path $y^i(t)$, we have seen that this limit can be reached without problems only in the case $p \leq 1$, which corresponds to the absence of cosmological horizons. On the other hand, the case of non-trivial horizons, $p > 1$, implies ultraviolet divergences, which prevent one from taking this limit in a way consistent with conformal invariance of the underlying σ model. In such a case, the operators are relevant, with finite anomalous dimensions $-\epsilon^2/2$, and thus Liouville dressing is required [8, 5]. This is the topic of the next subsection.

3.2 Cosmological Horizons and Liouville Dressing

In this subsection we shall discuss Liouville dressing of the relevant recoil deformations [5]. There are two ways one can proceed in this matter. The first, concerns dressing of the boundary operators (1)

$$V_{L,\text{boundary}} = \int_{\partial\Sigma} e^{\alpha_i \varphi} y_i(t) \partial_n X^i, \qquad \alpha_i = -\frac{Q}{2} + \sqrt{\frac{Q^2}{2} + (1 - h_i)} \tag{47}$$

where h_i is the boundary conformal dimension, and Q^2 is the induced central charge deficit on the boundary of the world-sheet.

In a similar spirit to the flat target-space case [3], the rate of change of Q^2 with respect to world-sheet scale $\mathcal{T} \sim \epsilon^{-2}$ is given by means of Zamolodchikov C-theorem [15], and it is found to be of order [5] $v_i^2\,\epsilon^4$, as being proportional to the square of the renormalization-group β^i functions ($i = v_i$): $\frac{\partial Q^2}{\partial \mathcal{T}} \propto -\beta^i \mathcal{G}_{ij} \beta^j$, where $\mathcal{G}_{ij} = \delta_{ij} + \ldots$, is the Zamolodchikov metric in coupling constant space. This implies that $Q^2(t) = Q_0^2 + \mathcal{O}(\epsilon^2)$, where Q_0^2 is constant.

We shall distinguish two cases for Q_0. The first concerns the case where $Q_0 \neq 0$ (and by appropriate normalization may be assumed to be of order $\mathcal{O}(1)$). This is the case of strings living in a non-critical space time dimension. The other pertains to the case where the only source of non-criticality is the impulse deformation, i.e. $Q_0 = 0$. In the former case, one has a Liouville dimension $\alpha_i \sim \epsilon^2$, while in the latter $\alpha_i \sim \epsilon$. In *both cases* however, $\epsilon \sim \frac{1}{t}$, where t is the target time.

In the second method [5], one dresses by the Liouville field the bulk operator (38), i.e.

$$V_{L,\text{bulk}} = \int_\Sigma e^{\alpha_i \varphi} \partial_\alpha \left(y_i(t) \partial^\alpha X^i \right), \qquad \alpha_i = -\frac{Q}{2} + \sqrt{\frac{Q^2}{2} + (2 - \Delta_i)} \tag{48}$$

where Δ_i is the conformal dimension of the bulk operator. The central charge deficit Q is of the same order $Q^2 = Q_0^2 + \mathcal{O}(\epsilon^2)$ as in the boundary case, which implies again that $\alpha_i \sim \epsilon^2$ if $Q_0 \neq 0$, and $\alpha_i \sim \epsilon$ if $Q_0 = 0$. An interesting question, which we shall answer in the affirmative below, concerns the equivalence between these two approaches either at the fixed point ($\epsilon \to 0$), or close to it ($\epsilon \neq 0$ but small).

We commence our analysis by first looking at the boundary operator (47). We may rewrite it as a bulk operator and then manipulate it as follows:

$$V_{L,\text{boundary}} = \int_\Sigma \partial_\alpha \left(e^{\alpha_i \varphi} y_i(t) \partial^\alpha X^i\right) = \\ = \int_\Sigma \alpha_i e^{\alpha_i \varphi}\, y_i(t) \partial_\alpha \varphi \partial^\alpha X^i + \int_\Sigma e^{\alpha_i \varphi} \dot{y}_i(t) \partial_\alpha t \partial^\alpha X^i + \int_\Sigma e^{\alpha_i \varphi} y_i(t) \partial^2 X^i \tag{49}$$

For the bulk operator (48) one has:

$$V_{L,\text{bulk}} = \int_\Sigma \partial_\alpha \left(e^{\alpha_i \varphi} y_i(t) \partial^\alpha X^i\right) - \int_\Sigma \alpha_i e^{\alpha_i \varphi}\, y_i(t) \partial_\alpha \varphi \partial^\alpha X^i = \\ = \int_{\partial\Sigma} e^{\alpha_i \varphi} y_i(t) \partial_n X^i - \int_\Sigma \alpha_i e^{\alpha_i \varphi}\, y_i(t) \partial_\alpha \varphi \partial^\alpha X^i \tag{50}$$

The logarithmic algebra, as discussed in [3] and above, implies a non-trivial infrared fixed point, which in the case $Q_0 \neq 0$ is determined by $\varphi_0 = \epsilon^{-2} \sim \ln(L/a)^2 \to \infty$, where φ_0 is the Liouville field world-sheet zero mode. Thus, $\alpha_i \varphi_0$ is finite as $\epsilon \to 0^+$. Therefore, as expected from the restoration of the conformal invariance by means of the Liouville dressing, one can now take safely the infra-red limit $\epsilon \to 0^+$ in the above expressions. It is then easy to see that one is left *in both cases* with the metric (43), thereby proving the equivalence of both approaches at the infrared fixed point.

In the case $Q_0 = 0$, the running central charge deficit $Q^2 = \mathcal{O}(\epsilon^2)$. Recalling [8] that the above formulæ imply a rescaling of the Liouville mode by $Q \sim \epsilon$, so as to have a canonical kinetic σ-model term [3], and that in this case it is the φ_0/Q which is identified with $\ln(L/a)^2 \sim \epsilon^{-2}$ as pertaining to the covariant world-sheet cutoff, one observes that again $\alpha_i\, \varphi$ is finite as $\epsilon \to 0^+$, and hence similar conclusions are reached concerning the equivalence of the two methods of Liouville dressing of the impulse operator.

This equivalence is also valid *close to*, but not exactly at, the infrared fixed point, as we demonstrate now. To this end, we discuss the two cases $Q_0 = 0$ and $Q_0 \neq 0$ separately.

Consider first the case $Q_0 \neq 0$. In this case $\alpha_i \sim \epsilon^2$, $\varphi_0 \sim \epsilon^{-2}$ and hence $\alpha_i \varphi_0 \sim \epsilon t = \text{const.}$ We identify now the Liouville direction φ with that of the target time [9, 5]. Given that $t \sim \frac{1}{\epsilon}$ this implies that $\varphi \sim t^2$. Under this identification we observe [5] in both cases (49), (50) that the terms $e^{\alpha \varphi_0}$, and the exponential factors $e^{-\epsilon\ (t-t_0)}$ appearing in the regulated Θ functions [3] are all of order one.

From these considerations one obtains an induced non-diagonal metric element G_{0i}, which in the case (49) is

$$G_{0i} d\varphi dX^i = \left(v_i\ \alpha_i t^{2p} + v_i a^2(t_0)\right)\ d(t^2) dX^i \simeq v_i\ \alpha_i t^{2p} d(t^2) dX^i\ , \qquad t \gg t_0 \tag{51}$$

and in the case (50):

$$G_{0i} d\varphi dX^i \simeq -v_i\ \alpha_i t^{2p} d(t^2) dX^i\ , \qquad t \gg t_0 \tag{52}$$

We then observe that, up to an irrelevant sign, the two results are equivalent in the regime of large t, where our perturbative string (world-sheet) analysis is valid, thereby proving the equivalence of the two ways of Liouville dressing even away from the fixed point.

Under the fact that one identifies $\epsilon^{-1} = t - t_0 \sim t \gg t_0$, the non-diagonal element of the spacetime metric becomes:

$$G_{0i} \sim v_i t^{2p-1} \tag{53}$$

We remind the reader that we analyze here the case with horizon, which implies $p > 1$.

It is convenient now to diagonalize the metric, which implies the following line element

$$ds^2 = -\frac{v_i^2}{a_0^2} t^{2p-2} dt^2 + a_0^2 t^{2p} (dX^i)^2 \tag{54}$$

[3] Notice that this rescaling becomes a trivial one in the case where $Q_0 \neq 0$.

By redefining the time coordinate to $t' = \frac{v_i}{a_0 p} t^p$ one obtains the induced line element:

$$ds^2 = -(dt')^2 + \frac{a_0^4\, p^2}{v_i^2} (t')^2\, (dX^i)^2, \qquad t \gg t_0 \tag{55}$$

From (20), we thus observe that the induced metric has *no horizon*, and no cosmic acceleration. In other words a recoiling D-particle, embedded in a space time which initially appeared to have an horizon, back reacted in such a way so as to remove it! Equivalently, we may say that recoiling D-particles are consistent only in spacetimes without cosmological horizons.

Similar conclusions are reached in the case $Q_0 = 0$. In that case, $\varphi/Q \sim \epsilon^{-2}$, as explained above, and since $Q \sim \epsilon \sim \frac{1}{t}$, one now has that $\varphi \sim t$. Again, the exponential terms $e^{\alpha_i \varphi}$, $\alpha_i \sim \epsilon$, and those coming from the regulated $\Theta_\epsilon(t)$ are of order one. Evidently, the induced non-diagonal metric has the same form (53) as in the case with $Q_0 \neq 0$, and one can thus repeat the previous analysis, implying removal of the cosmological horizon and stopping of cosmic acceleration.

The reader must have noticed that the same conclusion is reached already at the level of the metric (54), before the time transformation, once one interprets the coefficient of the $(dt)^2$ as a time-dependent light velocity. The fact that such situations arise 'suddenly', after a time moment t_0, might prompt the reader to draw some analogy with the scenaria of time-dependent light velocity, involving some sort of phase transitions at a certain moment in the (past) history of our Universe [16]. In our case, as we have seen, one can perform (at late times) a change in the time coordinate in order to arrive at a RW metric (55) [4].

The removal of the horizon would seem to imply from a field-theoretic point of view that one can define asymptotic states and thus a proper S-matrix. However, in the context of Liouville strings, with the Liouville mode identified with the time [9], there is no proper S-matrix, independently of the existence of horizons [12]. This has to do with the structure of the correlation functions of vertex operators in this construction, which are defined over steepest-descent closed time-like paths in a path-integral formalism, resembling closed-time paths of non-equilibrium field theory [9, 12]. In such constructions one can define properly only a (non factorizable) superscattering matrix $\$ \neq S\, S^\dagger$.

4 Conclusions

In this work we have analyzed in some detail the problem of impulse(recoil)-induced motion of a heavy D-particle in a Robertson-Walker spacetime, at large times t after the moment of impact. We have shown, that for RW spacetimes with scale factors $\sim t^p$, there is an order $[2p]$-logarithmic algebra, involving a group of impulse operators, which are relevant from a world-sheet renormalization group point of view.

A detailed study of how one can approach the non-trivial infrared fixed point is given. In the case where $p > 1$, which is physically characterized by the presence of cosmological horizons, one encounters world-sheet divergences. A proper subtraction of such divergences is subtle, and a detailed discussion of how this can be done has been presented. The fact that away from the fixed point the deformed theory is plagued by relevant deformations, of anomalous dimension which itself depends on the world-sheet renormalization-group scale, implies the need for Liouville dressing.

Such a dressing results in the interesting possibility of identifying the Liouville mode with the target time, in which case one has a formal description -in terms of conformal field theory methods on the world sheet - of back reaction effects of the recoiling D-particle on the surrounding space time. It is interesting to notice that the effect is equivalent to a 'phase transition' at the moment of impact, in which there is induced a time-varying speed of light, effectively leading to the removal of the initial cosmological horizon, and the eventual stopping of the acceleration of the Universe.

From a field-theoretic view point, this would imply that in such models proper asymptotic states, and thus an S-matrix, could be defined. However, from our stringy point of view, the

[4]There is a slight point to which we would like to draw the reader's attention. This regards the fact that such transformations depend on the recoil velocity, and thus on the energy content of the matter incident on the D-particle. In case one has a 'foam' situation [5], in which several incident particles interact with collections of D-particles, which are virtual quantum excitations of the string/brane vacuum, it is unclear how the present results are modified, and hence it might be that one cannot perform simultaneous transformations to diagonalize the metric, thereby obtaining non-trivial refractive indices [17]. Such issues fall beyond the scope of the present article.

definition of an S-matrix is still a complicated issue, since the underlying theory is of Liouville (non-equilibrium) type [12]. Whether such toy models are of relevance to realistic stringy cosmologies remains to be seen. Nevertheless, we believe that the results presented here, although preliminary, are of sufficient interest to prompt further studies along the directions suggested in this work.

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